3/31/17

"In order to understand the value in a shortcut, one must have taken the long road first."

-Anonymous

HW: "Exponential Modeling Day 2" homework section

Quarter Test on Wednesday 4/5

AIM: Exponential Modeling Cont.

Warm Up:

Percents combine in strange ways that don't seem to make sense at first. It would seem that if a population grows by 5% per year for 10 years, then it should grow in total by 50% over a decade. But this isn't true. Start with a population of 100. If it grows at 5% per year for 10 years, what is its population after 10 years? What percent growth does this represent?

$A = P(1+r)^t$

$A = 100(1+.05)^{10}$

$A = 162.889$

1.62889

$\underbrace{162.89\%}_{63\%}$

Percent growth $\approx 63\%$

P named rate r

***Exercise* #1:** A person invests $500 in an account that earns a **nominal yearly interest rate** of 4%.

(a) How much would this investment be worth in 10 years if the **compounding frequency** was once per year? Show the calculation you use.

t

\# of times per year

$A = P(1+r)^t$

$A = 500(1+.04)^{10}$

$A = \$740.12$

(b) If, on the other hand, the interest was applied four times per year (known as quarterly compounding), why would it not make sense to multiply by 1.04 each quarter?

No b/c 4% is the yearly rate, not the quarterly rate.

(c) If you were told that an investment earned 4% per year, how much would you assume was earned per quarter? Why?

4 quarters in a year

$\frac{4\%}{4} = 1\%$ per quarter

(d) Using your answer from part (c), calculate how much the investment would be worth after 10 years of quarterly compounding? Show your calculation.

4 times each year

$A = P(1+r)^t$ $4 \times 10 = 40$ times total

$A = 500(1+.01)^{40}$

$A = \$744.43$

So, the pattern is fairly straightforward. For a **shorter compounding period**, we get to **apply the interest more often**, but at a **lower rate**.

P

***Exercise* #2:** How much would $1000 invested at a nominal 2% yearly rate, compounded monthly, be worth in 20 years? Show the calculations that lead to your answer.

(1) $1485.95

(2) $1491.33

(3) $1033.87

(4) $1045.32

12 times per year

$\frac{.02}{12} \leftarrow$ monthly rate

$t = 20 \times 12 = 240$

$A = P(1+r)^t$

$A = 1000\left(1+\frac{.02}{12}\right)^{240}$

$\boxed{A = \$1491.33}$

This pattern is formalized in a classic formula from economics that we will look at in the next exercise.

***Exercise* #3:** For an investment with the following parameters, write a formula for the amount the investment is worth, A, after t-years.

P = amount initially invested

r = nominal yearly rate

n = number of compounds per year

$$\boxed{A(t) = P\left(1+\frac{r}{n}\right)^{nt}}$$

actual rate

***Exercise* #4:** An investment with a nominal rate of 5% is compounded at different frequencies. Give the **effective** yearly rate, accurate to two decimal places, for each of the following compounding frequencies. Show your calculation.

(a) Quarterly

1 year

$$\left(1+\frac{.05}{4}\right)^{4\cdot 1}$$

$$1\boxed{.0509}$$

5.09%

(b) Monthly

$$\left(1+\frac{.05}{12}\right)^{12\cdot 1}$$

$$1\underbrace{.05116}$$

5.12%

(c) Daily

$$\left(1+\frac{.05}{365}\right)^{365}$$

$$1\underbrace{.05126}$$

5.13%

We could compound at smaller and smaller frequency intervals, eventually compounding all moments of time. In our formula from *Exercise* #3, we would be letting n approach infinity. Interestingly enough, this gives rise to **continuous compounding** and the use of the natural base **e** in the famous **continuous compound interest formula.**

CONTINUOUS COMPOUND INTEREST

For an initial principal, P, compounded continuously at a nominal yearly rate of r, the investment would be worth an amount A given by:

$$A(t) = Pe^{rt}$$

***Exercise* #5:** A person invests $350 in a bank account that promises a nominal rate of 2% continuously compounded.

P

(a) Write an equation for the amount this investment would be worth after t-years.

$$A = 350e^{.02t}$$

(b) How much would the investment be worth after 20 years?

t

$$A = 350e^{.02(20)}$$

$$\boxed{A = \$522.14}$$

(c) Algebraically determine the time it will take for the investment to reach $400. Round to the nearest tenth of a year.

A

$$\frac{400}{350} = \frac{350e^{.02t}}{350}$$

$$\frac{400}{350} = e^{.02t}$$

$$\frac{.02t}{.02} = \frac{\ln \frac{400}{350}}{.02}$$

$$t = 6.7 \text{ years}$$

(d) What is the effective annual rate for this investment? Round to the nearest hundredth of a percent.

$$e^{.02(1)} = 1.0202$$

$$2.02\%$$